PART 2: A MECHANISM THAT ORGANIZES CLOUD PATTERNS

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ABSTRACT

The shape of the frontal or inversion surface over a lower moist layer with geostrophic wind balance is shown to be curved, and a steady-state distribution with a discontinuity in the shape of the surface is shown to be possible. As the resulting configuration of stationary ridges on the inversion conforms with many features of cloud streets, it is suggested that the mechanism may play an important role in the organization of cloud patterns. Presentation of the complete theory must await a more elaborate theoretical-descriptive paper.

This brief note on the existence of stationary ridges on an inversion over moist air is a preliminary write-up of the backbone of a much more elaborate theoretical-descriptive paper by the authors of this memorial discussion of cloud bands. The universal nature of a Margules equation for moist air and the attractive theory of stationary finite waves encourage this early publication. We are looking for a clear-cut irrefutable example of such ridges to include in the completed paper. We already have many plausible examples, so we are confident of ultimate publication. Since we are working without funding, the full paper may not appear for some years.

Assuming steady-state conditions, let us examine the interface between two uniform air masses, the lower one completely mixed—with an adiabatic lapse rate up to the condensation level, H_c, and with a moist adiabatic lapse rate from there to the top of the layer. See figure 11.

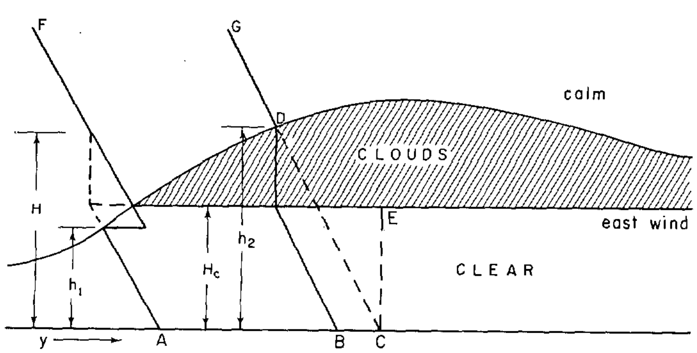

FIGURE 11.—Idealized soundings through completely mixed lower moist layer surmounted by an unsaturated upper air mass. CD = dry adiabat, CE = moist adiabat; AF = sounding with interface at h_1; BDG = sounding with interface at h_2; y = distance north from $y=0$ where the interface meets the ground.

The height, H, is defined as the height to which a parcel of the lower moist air mass would have to be lifted in order for its temperature to equal that of the parcel of the upper air mass in contact with it. In this simplified case let us assume (although some of these assumptions are unnecessary for full development of the problem) that the upper air mass is unsaturated and calm and that there is a uniform east wind in the lower air mass. Moreover, we assume that the cold lower air mass and the east winds cover a vast expanse and we seek a steady-state solution of the equations of motion.

Solving the geostrophic wind equations with the assumption that the pressure gradient depends on the weight of air below the interface we obtain

$$-fu = B\frac{\partial h}{\partial y} \tag{1}$$

where h is the height of the interface and $B = g\frac{\rho-\rho'}{\rho}$ where ρ is the density of the lower moist air at the interface and ρ' is the density of the upper dry air at the interface. When h is below h_c so that all of the lower air is unsaturated, then we have $B=B_0$, a constant. When h is above H_c so that the moist air is saturated we have a variable B, namely, $B=B_0\frac{H-h}{H-H_c}$. Note that at H_c, $B=B_0$ in both formulas and that at H, $B=0$.

Northward from where the interface is at H_c we have,

$$\frac{\partial h}{\partial y} = -\frac{fu}{B} = -\frac{fu(H-H_c)}{B_0(H-h)} \tag{2}$$

The solution of equation (2) for the height h of a saturated stable layer is:

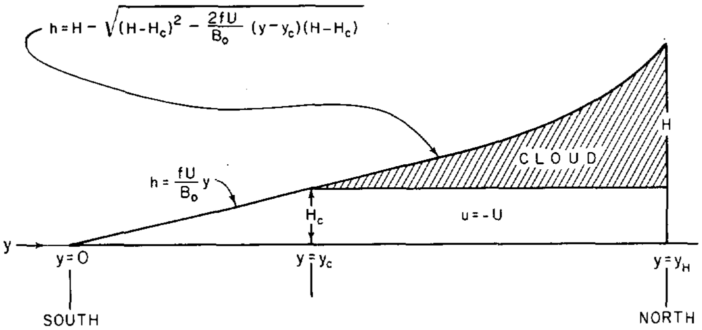

FIGURE 12.—Height h of frontal surface northward from $y=0$, where it meets the ground, to $y=y_H$.

$$(H-h)^2=\frac{2fu(H-H_c)}{B_0}\,y+C \tag{3}$$

When $h=H_c$, equation (3) can be written

$$(H-H_c)^2=\frac{2fuy_c}{B_0}\,(H-H_c)+C=2H_c(H-H_c)+C$$

so that $C=(H-H_c)^2-2H_c(H-H_c)$. Substituting in equation (3)

$$(H-h)^2=\frac{2fu(H-H_c)}{B_0}\,(y-y_c)+(H-H_c)^2$$

or

$$h=H-\sqrt{(H-H_c)^2+\frac{2fu}{B_0}\,(y-y_c)\,(H-H_c)} \tag{4}^*$$

The frontal surface from $y=0$ northward has the shape indicated in figure 12. Now we note that there is nothing startling about this interface shape as long as the radical in equation (8) is positive. However, despite the constant geostrophic flow, there will be a value for y, y_H, above which the radical in equation (8) will be negative. That is,

$$y_H=y_c-\frac{B_0}{2fu}\,(H-H_c)$$

is the *northern* limit of the *continuous* interface surface described above.

One possible configuration for a discontinuous interface between two air masses (in both of which the flow is geostrophic) is diagramed in figure 13.

The equation,

$$B_0(H-h)\,\frac{\partial h}{\partial y}=\text{const.}$$

is true even though h is discontinuous because the geostrophic wind is related to the derivative of the height

*This is a sort of Margules equation.

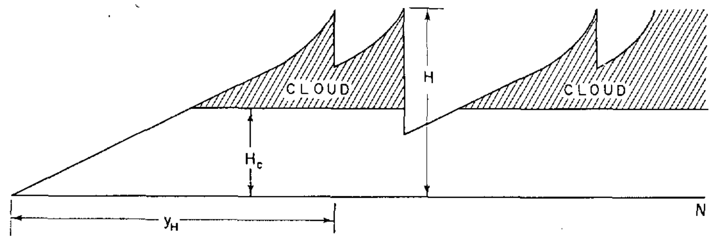

FIGURE 13.—Cross-section of a discontinuous interface in a constant geostrophic flow.

with respect to y and not to the height itself. If $\partial h/\partial y$ is continuous, equation (3) is still valid even if h is discontinuous. This is one of those fine points of mathematics which mathematicians often feel have no physical significance, but this particular fine point is significant—physically.

In figure 13 the discontinuity of the interface when $h=H$ can be nearly stationary because its speed is approximately $\sqrt{Bh}$ and when $h=H$, $B=0$ in our model. So far we have assumed the upper air mass at the interface is calm. If there were a geostrophic wind in the upper air mass, the cloud pattern would move with the wind. The cloud streets would then be parallel to the wind shear. We would like to emphasize that this development calls for bands parallel to the wind shear in the stable layer moving with the component of the upper wind normal to the band. (The cloud elements in the band would be expected to have a component of motion directly opposed to the shear vector.)

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